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A LETTER

TO

WILLIAM WHEWELL,

PROF. OF MORAL PHILOSOPHY IN THE UNIVERSITY OF CAMBRIDGE, ENG.,

IN REPLY TO CERTAIN ALLEGATIONS AND ARGUMENTS ADVANCED IN A

PAMPHLET ENTITLED A

DEMONSTRATION THAT ALL MATTER IS HEAVY.

BY ROBERT HARE, M. D.

PROF. OF CHEMISTRY IN THE UNIVERSITY OF PENNSYLVANIA.

A LETTER, &c.

1. *Dear Sir*—I thank you for your kind attention in sending me a copy of your pamphlet entitled a "*Demonstration that all Matter is Heavy*," comprising a communication made to the Cambridge Philosophical Society.

2. I conceive that to demonstrate that all matter is heavy, is, in other words, to prove that all matter is endowed with attraction of gravitation, or that general property which, when it causes bodies to tend towards the centre of the earth, is called weight. Hence to assert that all matter is heavy, is no more than to say, that attraction of gravitation exists between all or any masses of matter.*

3. You say, "it may be urged that we have no difficulty in conceiving of matter which is not heavy." I have no hesitation in asserting, that there should be no difficulty in entertaining such a conception; since I cannot understand why any two masses may not be as readily conceived to *repel* as to *attract* each other, or *neither to attract nor to repel.* Is it not easier to imagine two remote masses indifferent to each other, than that they act upon each other? Is any thing more difficult to understand than that a body can act where it is not?

4. It is also mentioned by you, that it may be urged "*that inertia and weight are two separate properties of matter.*" Now I will not only urge, but also, with all due deference, will undertake to show, that the existence of inertia may as well be proven, and its quantity estimated, by means of repulsion as by means of attraction.

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* We have thought that Dr. Hare's letter would be better understood by our readers, if we republished the "Demonstration" of Prof. Whewell, as it has probably been seen by few persons in America. It will accordingly be found in full at the end of Dr. Hare's letter.—Eds.

5. Suppose two bodies, A and B, to be endowed with reciprocal attraction; or, in other words, to gravitate towards each other. Being placed at a distance, and then allowed to approach, if, after any given time, it were found that they had moved severally any ascertained distances, evidently their relative inertias would be considered as inversely as those distances.

6. In the next place, let us suppose two bodies, X and Y, endowed with the opposite force of reciprocal repulsion, to be placed in proximity, and then allowed to fly apart. The distances run through by them severally, being, at any given time, determined, might not their respective inertias be taken to be inversely as those distances; so that the question would be as well ascertained in this case, as in that above stated in which gravitation should be resorted to as the test?

7. It seems to me that this question is sufficiently answered, in the affirmative, in your second paragraph, page 7, (p. 269,) in which you allege, that "*one body has twice as much inertia as another, if when the same force acts upon it for the same time, it acquires but half the velocity. This is the fundamental conception of inertia.*"

8. In the third paragraph, fourth page, (p. 261,) you say, "*that the quantity of matter is measured by those sensible properties of matter which undergo quantitative addition, subtraction and division, as the matter is added, subtracted or divided, the quantity of matter cannot be known in any other way; but this mode of measuring the quantity of matter in order to be true at all, must be true universally.*"

9. Also your fourth paragraph, fifth page, (p. 268,) concludes with this allegation, "*and thus we have proved, that if there be any kind of matter which is not heavy, the weight can no longer avail us, in any case to any extent, as the measure of the quantity of matter.*"

10. In reply to these allegations let me inquire, cannot a matter exist of which the sensible properties do not admit of being measured by human means? Because some kinds of matter can be measured by "those sensible qualities which undergo quantitative addition, subtraction and division," does it follow that there may not be matter which is incapable of being thus measured? And wherefore would the method of obtaining phi-

losophical truth be "futile" in the one case, because inapplicable in the other? Because the inertias of A and B have been discovered, by means of their gravitation, does it follow that the inertias of X and Y cannot be discovered by their self-repellent power? Why should the inapplicability of gravitation in the one case render its employment futile in the other?

11. It is self-evident, that matter without weight cannot be estimated by weighing, but I deny that on that account such weightless matter may not be otherwise estimated. The inertias of A and B cannot be better measured by gravitation than those of X and Y by repulsion, as already shown.

12. You seem to infer, in paragraph second, page sixth, (p. 268,) that we should be equally destitute of the means of measuring matter accurately, "*were any kind of matter heavy indeed, but not so heavy, in proportion to its quantity of matter, as other kinds.*"

13. If in the case of all matter weight be admitted to be the only measure of quantity, it were inconsistent to suppose any given quantity of matter, of any one kind, to have less weight than an equal quantity of another kind; but upon what other than a conventional basis is it to be assumed, that there is more matter in a cubic inch of platinum than in a cubic inch of tin; in a cubic inch of mercury than in a cubic inch of iron? Judging by the chemical efficacy of the masses, although the weight of mercury is to that of iron as 13.6 is to 8, there are more equivalents of the latter than the former in any given bulk, since by weight twenty-eight parts of iron are equivalent to two hundred and two parts of mercury.

14. Weight is one of the properties of certain kinds of matter, and has been advantageously resorted to, in preference to any other property, in estimating the quantity of the matter to which it appertains. Nevertheless, measurement by bulk is found expedient or necessary in many cases. But may we not appeal to any general property which admits of being measured or estimated? Faraday has inferred that the quantity of electricity, is as the quantity of gas which it evolves. Light has been considered as proportional in quantity to the surface which it illuminates with a given intensity at a certain distance. The quantity of caloric has been held to be directly as the weight of water which it will render aëriform; and has also been estima-

ted by the degree of its expansive or thermometric influence. What scale-beam is more delicate than the thermoscope of Melloni?

15. In the last paragraph but one, seventh page, (p. 270,) you suggest, that "*perhaps some persons might conceive that the identity of weight and inertia is obvious at once, for both are merely resistance to motion; inertia, resistance to all motion, or change of motion; weight, resistance to motion upwards.*"

16. I am surprised that you should think the opinion of any person worthy of attention, who should entertain so narrow a view of weight, as antagonist of momentum, as that above quoted, "*that it is a resistance to motion upwards.*" Agreeably to the definition, given at the commencement of the letter, weight, in its usual practical sense, is only one case of the general force which causes all ponderable masses of matter to gravitate towards each other, and which is of course liable to resist any conflicting motion, whatever may be the direction. When in the form of solar attraction, it overcomes that inertia of the planets which would otherwise cause them to leave their orbits, does gravitation "*resist motion upwards?*"

17. In the next paragraph you allege, that "*there is a difference in these two kinds of resistance to motion. Inertia is instantaneous, weight is continuous resistance.*"

18. It is to this allegation I object, that as you have defined inertia to be "*resistance to motion, or to change of motion,*" it follows that it can be instantaneous only where the impulse which it resists is instantaneous. It cannot be less continuous than the force by which it is overcome.

19. Gravity has been considered as acting upon falling bodies by an infinity of impulses, each producing an adequate acceleration; but to every such accelerating impulse, producing of course a "*change of motion,*" will there not be a commensurate resistance from inertia? and the impulses and resistances being both infinite, will not one be as continuous as the other?

20. I have already adverted to inertia as the continuous antagonist of solar attraction in the case of revolving planets.

21. Agreeably to Mossotti, the creation consists of two kinds of matter, of which the homogeneous particles are mutually repellent, the heterogeneous mutually attractive. Consistently with

this hypothesis, *per se*, any matter must be imponderable ; being endowed with a property the very opposite of attraction of gravitation. This last mentioned property exists between masses consisting of both kinds of particles, so far as the attraction between the heterogeneous atoms predominates over the repulsion between those which are homogeneous. It would follow from these premises, that all matter is ponderable or otherwise, accordingly as it may be situated.

22. Can the ether by which, according to the undulatory theory, light is transmitted, consist of ponderable matter? Were it so, would it not be attracted about the planets with forces proportioned to their weight, respectively? and becoming of unequal density, would not the diversity in its density, thus arising, affect its undulations, as the transmission of sound is influenced by any variations in the density of the aëriform fluid by which it is propagated?

With esteem,

I am yours truly,

ROBERT HARE.

Demonstration that all Matter is Heavy. By the Rev. WILLIAM WHEWELL, B. D., *Fellow of Trinity College and Professor of Moral Philosophy.* [Read February 22d, 1841.]

"The discussion of the nature of the grounds and proofs of the most general propositions which the physical sciences include, belongs rather to metaphysics than to that course of experimental and mathematical investigation by which the sciences are formed. But such discussions seem by no means unfitted to occupy the attention of the cultivators of physical science. The ideal, as well as the experimental side of our knowledge, must be carefully studied and scrutinized, in order that its true import may be seen ; and this province of human speculation has been perhaps of late unjustly depreciated and neglected by men of science. Yet it can be prosecuted in the most advantageous manner by them only : for no one can speculate securely and rightly respecting the nature and proofs of the truths of science without a steady possession of some large and solid portions of such truths. A man must be a mathematician, a mechanical philosopher, a natural historian, in order that he may philosophize well concerning mathematics, and mechanics,

and natural history; and the mere metaphysician who without such preparation and fitness sets himself to determine the grounds of mathematical or mechanical truths, or the principles of classification, will be liable to be led into error at every step. He must speculate by means of general terms, which he will not be able to use as instruments of discovering and conveying philosophical truth, because he cannot, in his own mind, habitually and familiarly, embody their import in special examples.

"Acting upon such views, I have already laid before the Philosophical Society of Cambridge essays on such subjects as I here refer to; especially a memoir "On the Nature of the Truth of the Laws of Motion," which was printed by the Society in its Transactions. This memoir appears to have excited in other places, notice of such a kind as to shew that the minds of many speculative persons are ready for and inclined towards the discussion of such questions. I am therefore the more willing to bring under consideration another subject of a kind closely related to the one just mentioned.

"The general questions which all such discussions suggest, are (in the existing phase of English philosophy) whether certain proposed scientific truths, (as the laws of motion,) be *necessary* truths; and if they are necessary, (which I have attempted to shew that in a certain sense they are,) *on what ground* their necessity rests. These questions may be discussed in a general form, as I have elsewhere attempted to shew. But it may be instructive also to follow the general arguments into the form which they assume in special cases; and to exhibit, in a distinct shape, the incongruities into which the opposite false doctrine leads us, when applied to particular examples. This accordingly is what I propose to do in the present memoir, with regard to the proposition stated at the head of this paper, namely, that *all matter is heavy*.

"At first sight it may appear a doctrine altogether untenable to assert that this proposition is a necessary truth: for it may be urged, we have no difficulty in conceiving matter which is not heavy; so that matter without weight is a conception not inconsistent with itself; which it must be if the reverse were a necessary truth. It may be added, that the possibility of conceiving matter without weight was shewn in the controversy which ended in the downfall of the phlogiston theory of chemical composition; for some of the reasoners on this subject asserted phlogiston to be a body with positive levity instead of gravity, which hypothesis, however false, shews that such a supposition is possible. Again, it may be said that *weight* and *inertia* are two separate properties of matter; that mathematicians measure the quantity of matter by the inertia, and that we learn by experiment only that the

weight is proportional to the inertia; Newton's experiments with pendulums of different materials having been made with this very object.

"I proceed to reply to these arguments. And first, as to the possibility of conceiving matter without weight, and the argument thence deduced, that the universal gravity of matter is not a necessary truth, I remark, that it is indeed just to say that we cannot even distinctly conceive the contrary of a necessary truth to be true; but that this impossibility can be asserted only of those perfectly distinct conceptions which result from a complete developement of the fundamental idea and its consequences. Till we reach this stage of developement, the obscurity and indistinctness may prevent our perceiving absolute contradictions, though they exist. We have abundant store of examples of this even in geometry and arithmetic; where the truths are universally allowed to be necessary, and where the relations which are impossible, are also inconceivable, that is, not conceivable distinctly. Such relations, though not distinctly conceivable, still often appear conceivable and possible, owing to the indistinctness of our ideas. Who, at the first outset of his geometrical studies, sees any impossibility in supposing the side and the diagonal of a square to have a common measure? Yet they can be rigorously proved to be incommensurable, and therefore the attempt distinctly to conceive a common measure of them must fail. The attempts at the geometrical duplication of the cube, and the supposed solutions, (as that of Hobbes) have involved absolute contradictions; yet this has not prevented their being long and obstinately entertained by men, even of minds acute and clear in other respects. And the same might be shewn to be the case in arithmetic. It is plain, therefore, that we cannot, from the supposed possibility of conceiving matter without weight, infer that the contrary may not be a necessary truth.

"Our power of judging, from the compatibility or incompatibility of our conceptions, whether certain propositions respecting the relations of ideas are true or not, must depend entirely, as I have said, upon the degree of developement which such ideas have undergone in our minds. Some of the relations of our conceptions on any subject are evident upon the first steady contemplation of the fundamental idea by a sound mind: these are the *axioms* of the subject. Other propositions may be deduced from the axioms by strict logical reasoning. These propositions are no less *necessary* than the axioms, though to common minds their *evidence* is very different. Yet as we become familiar with the steps by which these ulterior truths are deduced from the axioms, *their* truth also becomes evident, and the contrary becomes inconceivable. When a person has familiarized himself with the first twenty-six

propositions of Euclid, and not till then, it becomes evident to him, that parallelograms on the same base and between the same parallels are equal; and he cannot even conceive the contrary. When he has a little further cultivated his geometrical powers, the equality of the square on the hypothenuse of a right-angled triangle to the squares on the sides, becomes also evident; the steps by which it is demonstrated being so familiar to the mind as to be apprehended without a conscious act. And thus, the contrary of a necessary truth cannot be distinctly conceived; but the incapacity of forming such a conception is a condition which depends upon cultivation, being intimately connected with the power of rapidly and clearly perceiving the connection of the necessary truth under consideration with the elementary principles on which it depends. And thus, again, it may be that there is an absolute impossibility of conceiving matter without weight; but then, this impossibility may not be apparent, till we have traced our fundamental conceptions of matter into some of their consequences.

"The question then occurs, whether we can, by any steps of reasoning, point out an inconsistency in the conception of matter without weight. This I conceive we may do, and this I shall attempt to shew.

"The general mode of stating the argument is this:—the quantity of matter is measured by those sensible properties of matter which undergo quantitative addition, subtraction and division, as the matter is added, subtracted and divided. The quantity of matter cannot be known in any other way. But this mode of measuring the quantity of matter, in order to be true at all, must be universally true. If it were only partially true, the limits within which it is to be applied would be arbitrary; and therefore the whole procedure would be arbitrary, and, as a method of obtaining philosophical truth, altogether futile.

"We may unfold this argument further. Let the contrary be supposed, of that which we assert to be true: namely, let it be supposed that while all other kinds of matter are heavy, (and of course heavy in proportion to the quantity of matter,) there is one kind of matter which is absolutely destitute of weight; as, for instance, phlogiston, or any other element. Then where this *weightless* element (as we may term it) is mixed with *weighty* elements, we shall have a compound, in which the weight is no longer proportional to the quantity of matter. If, for example, 2 measures of heavy matter unite with 1 measure of phlogiston, the weight is as 2, and the quantity of matter as 3. In all such cases, therefore, the weight ceases to be the measure of the quantity of matter. And as the proportion of the weighty and the weightless matter may vary in innumerable degrees in such compounds, the weight affords no criterion at all of the quantity of matter in them.

And the smallest admixture of the weightless element is sufficient to prevent the weight from being taken as the measure of the quantity of matter.

"But on this hypothesis, how are we to distinguish such compounds from bodies consisting purely of heavy matter? How are we to satisfy ourselves that there is not, in every body, some admixture, small or great, of the weightless element? If we call this element *phlogiston*, how shall we know that the bodies with which we have to do are, any of them, absolutely free from phlogiston?

"We cannot refer to the weight for any such assurance; for by supposition the presence and absence of phlogiston makes no difference in the weight. Nor can any other properties secure us at least from a very small admixture; for to assert that a mixture of 1 in 100 or 1 in 10 of phlogiston would always manifest itself in the properties of the body, must be an arbitrary procedure, till we have proved this assertion by experiment; and we cannot do this till we have learnt some mode of measuring the quantities of matter in bodies and parts of bodies; which is exactly what we question the possibility of, in the present hypothesis.

"Thus, if we assume the existence of an element, *phlogiston*, devoid of weight, we cannot be sure that every body does not contain some portion of this element; while we see that if there be an admixture of such an element, the weight is no longer any criterion of the quantity of matter. And thus we have proved, that if there be any kind of matter which is not heavy, the weight can no longer avail us, *in any case or to any extent*, as a measure of the quantity of matter.

"I may remark, that the same conclusion is easily extended to the case in which phlogiston is supposed to have absolute levity; for in that case, a certain mixture of phlogiston and of heavy matter would have no weight, and might be substituted for phlogiston in the preceding reasoning.

"I may remark also, that the same conclusion would follow by the same reasoning, if any kind of matter, instead of being void of weight, were heavy indeed, but not *so* heavy, in proportion to its quantity of matter, as other kinds.

"On all these hypotheses there would be no possibility of measuring quantity of matter by weight at all, in any case, or to any extent.

"But it may be urged, that we have not yet reduced the hypothesis of matter without weight to a contradiction; for that mathematicians measure quantity of matter, not by weight, but by the other property, of which we have spoken, inertia.

"To this I reply, that, practically speaking, quantity of matter is always measured by weight, both by mechanicians and chemists: and as we have proved that this procedure is utterly insecure in all cases, on the hypothesis of weightless matter, the practice rests upon a conviction that the hypothesis is false. And yet the practice is universal. Every experimenter measures quantity of matter by the balance. No one has ever thought of measuring quantity of matter by its inertia practically; no one has constructed a measure of quantity of matter in which the matter produces its indications of quantity by its motion. When we have to take into account the inertia of a body, we inquire what its weight is, and assume this as the measure of the inertia; but we never take the contrary course, and ascertain the inertia first in order to determine by that means the weight.

"But it may be asked, Is it not then true, and an important scientific truth, that the *quantity of matter* is measured by the *inertia*? Is it not true, and proved by experiment, that the *weight* is *proportional* to the *inertia*? If this be not the result of Newton's experiments mentioned above, what, it may be demanded, do they prove?

"To these questions I reply: It is true that quantity of matter is measured by the inertia, for it is true that inertia is as the quantity of matter. This truth is indeed one of the laws of motion. That weight is proportional to inertia is proved by experiment, as far as the laws of motion are so proved: and Newton's experiments prove one of the laws of motion, so far as any experiments can prove them, or are needed to prove them.

"That inertia is proportional to weight, is a law equivalent to that law which asserts, that when pressure produces motion in a given body, the velocity produced in a given time is as the pressure. For if the velocity be as the pressure, when the body is given, the velocity will be constant if the inertia also be as the pressure. For the inertia is understood to be that property of bodies to which, *ceteris paribus*, the velocity impressed is *inversely* proportional. One body has twice as much inertia as another, if, when the same force acts upon it for the same time, it acquires but half the velocity. This is the fundamental conception of *inertia*.

"In Newton's pendulum experiments, the pressure producing motion was a certain resolved part of the weight, and was proportional to the weight. It appeared by the experiments, that whatever were the material of which the pendulum was formed, the rate of oscillation was the same; that is, the velocity acquired was the same. Hence the inertia of the different bodies must have been in each case as the weight: and thus this assertion is true of all different kinds of bodies.

"Thus it appears that the assertion, that inertia is universally proportional to weight, is equivalent to the law of motion, that the velocity is as the pressure. The conception of inertia (of which, as we have said, the fundamental conception is, that the velocity impressed is inversely proportional to the inertia,) connects the two propositions so as to make them identical.

"Hence our argument with regard to the universal gravity of matter brings us to the above law of motion, and is proved by Newton's experiments in the same sense in which that law of motion is so proved.

"Perhaps some persons might conceive that the identity of weight and inertia is obvious at once; for both are merely resistance to motion;—inertia, resistance to all motion (or change of motion)—weight, resistance to motion upwards.

"But there is a difference in these two kinds of resistance to motion. Inertia is instantaneous, weight is continuous resistance. Any momentary impulse which acts upon a free body overcomes its inertia, for it changes its motion: and this change once effected, the inertia opposes any return to the former condition, as well as any additional change. The inertia is thus overcome by a momentary force. But the weight can only be overcome by a continuous force like itself. If an impulse act in opposition to the weight, it may for a moment neutralize or overcome the weight; but if it be not continued, the weight resumes its effect, and restores the condition which existed before the impulse acted.

"But weight not only produces rest, when it is resisted, but motion, when it is not resisted. Weight is measured by the reaction which would balance it; but when unbalanced, it produces motion, and the velocity of this motion increases constantly. Now what determines the velocity thus produced in a given time, or its rate of increase? What determines it to have one magnitude rather than another? To this we must evidently reply, *the inertia.* When weight produces motion, the inertia is the reaction which makes the motion determinate. The accumulated motion produced by the action of unbalanced weight is as determinate a condition as the equilibrium produced by balanced weight. In both cases the condition of the body acted on is determined by the opposition of the action and reaction.

"Hence inertia is the reaction which opposes the weight, when unbalanced. But by the conception of action and reaction, (as mutually determining and determined,) they are measured by each other: and hence the inertia is necessarily proportional to the weight.

"But when we have reached this conclusion, the original objection may be again urged against it. It may be said, that there must be some

fallacy in this reasoning, for it proves a state of things to be necessary when we can so easily conceive a contrary state of things. Is it denied, the opponent may ask, that we can readily imagine a state of things in which bodies have no weight? Is not the uniform tendency of all bodies in the same direction not only not necessary, but not even true? For they do in reality tend, not with equal forces in parallel lines, but to a centre with unequal forces, according to their position: and we can conceive these differences of intensity and direction in the force to be greater than they really are; and can with equal ease suppose the force to disappear altogether.

"To this I reply, that certainly we may conceive the weight of bodies to vary in intensity and direction, and by an additional effort of imagination may conceive the weight to vanish: but that in all these suppositions, even in the extreme one, we must suppose the rule to be universal. If *any* bodies have weight, *all* bodies must have weight. If the direction of weight be different in different points, this direction must still vary according to the *law of continuity;* and the same is true of the intensity of the weight. For if this were not so, the rest and motion, the velocity and direction, the permanence and change of bodies, as to their mechanical condition, would be arbitrary and incoherent: they would not be subject to mechanical ideas; that is, not to ideas at all; and hence these conditions of objects would in fact be inconceivable. In order that the universe may be possible, that is, may fall under the conditions of intelligible conceptions, we must be able to conceive a body at rest. But the rest of bodies (except in the absolute negation of all force) implies the equilibrium of opposite forces. And one of these opposite forces must be a *general* force, as weight, in order that the universe may be governed by general conditions. And this general force, by the conception of force, may produce motion, as well as equilibrium; and this motion again must be determined, and determined by general conditions; which cannot be, except the communication of motion be regulated by an inertia proportional to the weight.

"But it will be asked, Is it then pretended that Newton's experiment, by which it was intended to prove inertia proportional to weight, does really prove nothing but what may be demonstrated *à priori?* Could we know, without experiment, that all bodies,—gold, iron, wood, cork,—have inertia proportional to their weight? And to this we reply, that experiment holds the same place in the establishment of this, as of the other fundamental doctrines of mechanics. Intercourse with the external world is requisite for developing our ideas; measurement of phenomena is needed to fix our conceptions and to render them precise; but the result of our experimental studies is, that we reach a position in which

our convictions do not rest upon experiment. We learn by observation truths of which we afterwards see the necessity. This is the case with the laws of motion, as I have repeatedly endeavored to shew. The same will appear to be the case with the proposition, that bodies of different kinds have their inertia proportional to their weight.

"For bodies *of the same kind* have their inertia proportional to their weight, both quantities being proportional to the quantity of matter. And if we compress the same quantity of matter into half the space, neither the weight nor the inertia is altered, because these depend on the quantity of matter alone. But in this way we obtain a body of *twice the density;* and in the same manner we obtain a body of any other density. Therefore whatever be the density, the inertia is proportional to the quantity of matter. But the mechanical relations of bodies cannot depend upon any difference of *kind*, *except* a difference of density. For if we suppose any fundamental difference of mechanical nature in the particles or component elements of bodies, we are led to the same conclusion, of arbitrary, and therefore, impossible, results, which we deduced from this supposition with regard to weight. Therefore all bodies of different density, and hence, all bodies whatever, must have their inertia proportional to their weight.

"Hence we see, that the propositions, that all bodies are heavy, and that inertia is proportional to weight, necessarily follow from those fundamental ideas which we unavoidably employ in all attempts to reason concerning the mechanical relations of bodies. This conclusion may perhaps appear the more startling to many, because they have been accustomed to expect that fundamental ideas and their relations should be self-evident at our first contemplation of them. This, however, is far from being the case, as I have already shewn. It is not the *first*, but the most complete and developed condition of our conceptions which enables us to see what are axiomatic truths in each province of human speculation. Our fundamental ideas are necessary conditions of knowledge, universal forms of intuition, inherent types of mental development; they may even be termed, if any one chooses, results of connate intellectual tendencies; but we cannot term them *innate* ideas, without calling up a large array of false opinions. For innate ideas were considered as capable of composition, but by no means of simplification; as most perfect in their original condition; as to be found, if any where, in the most uneducated and most uncultivated minds; as the same in all ages, nations, and stages of intellectual culture; as capable of being referred to at once, and made the basis of our reasonings, without any special acuteness or effort: in all which circumstances the fundamental ideas of which we have spoken, are opposed to innate ideas so understood.

"I shall not, however, here prosecute this subject. I will only remark, that fundamental ideas, as we view them, are not only not innate, in any usual or useful sense, but they are not necessarily *ultimate* elements of our knowledge. They are the results of our analysis so far as we have yet prosecuted it; but they may themselves subsequently be analyzed. It may hereafter appear, that what we have treated as different fundamental ideas have, in fact, a connexion, at some point below the structure which we erect upon them. For instance, we treat of the mechanical ideas of force, matter, and the like, as distinct from the idea of substance. Yet the principle of measuring the quantity of matter by its weight, which we have deduced from mechanical ideas, is applied to determine the substances which enter into the composition of bodies. The idea of substance supplies the axiom, that the whole quantity of matter of a compound body is equal to the sum of the quantities of matter of its elements. The mechanical ideas of force and matter lead us to infer that the quantity both of the whole and its parts must be measured by their weights. *Substance* may, for some purposes, be described as that to which properties belong; *matter* in like manner may be described as that which resists force. The former involves the idea of permanent being; the latter the idea of causation. There may be some elevated point of view from which these ideas may be seen to run together. But even if this be so, it will by no means affect the validity of reasonings founded upon these notions, when duly determined and developed. If we once adopt a view of the nature of knowledge which makes necessary truth possible at all, we need be little embarrassed by finding how closely connected different necessary truths are; and how often, in exploring towards their roots, different branches appear to spring from the same stem. W. Whewell."

Grange, August 31, 1840.